# Elastic Scattering with CDCC Method

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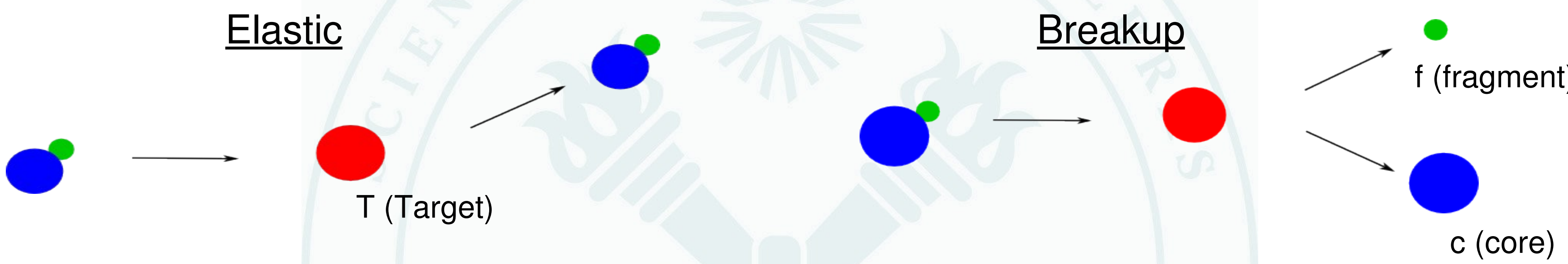
# Table of contents

# 1. Introduction

**Collision between two nuclei**




Goal : Differential elastic and breakup cross sections

Projectile : Nucleus with structure (core + fragment (2-body))

For example :
- halo nuclei
- $^{2}$H, $^{8}$B, $^{11}$Be, $^{17}$F, $^{6}$He (3-body), ...
- ...

Application : Low binding energy → importance of continuum

Example : $^{2}$H ← E = -2.22 MeV

# 2. CDCC method [1/4]

[G. Rawitscher, Phys. Rev. C 9 (1974)]

CDCC=Coupled Discretized Continuum Channel

Schrödinger's equation : $H\Psi(\vec{R},\vec{r}) = E\Psi(\vec{R},\vec{r})$

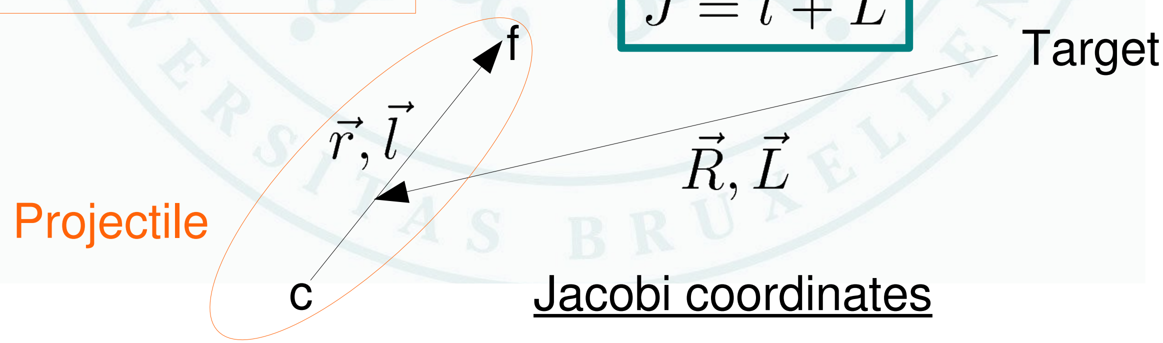
Hamiltonian : $H = T_r + V_{cf} + T_R + V_{cT}(\vec{R},\vec{r}) + V_{fT}(\vec{R},\vec{r})$

With the projectile (c+f)

$$H_0\Phi(\vec{r}) = \epsilon\Phi(\vec{r})$$

$$H_0 = T_r + V_{cf}(r)$$

$$\boxed{\vec{J} = \vec{l} + \vec{L}}$$




Jacobi coordinates

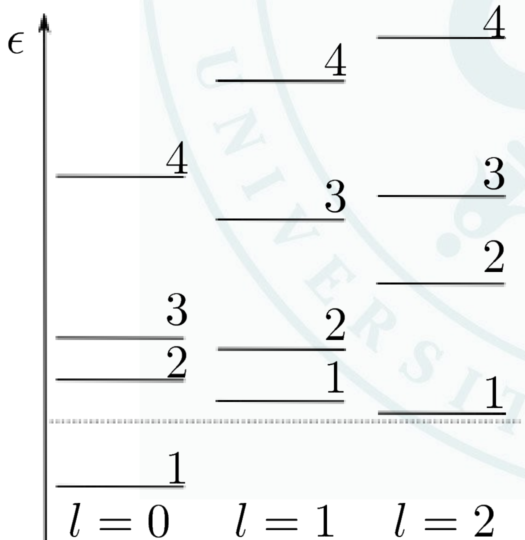
# 2. CDCC method [2/4]

## Principle of a Coupled-Channel method

$$\Psi(\vec{R},\vec{r}) = \underbrace{\Phi_0(\vec{r})\chi_0(\vec{R})}_{\epsilon<0 \text{ Bound state}} + \int \underbrace{\Phi_{\vec{k}}(\vec{r})\chi_{\vec{k}}(\vec{R})d\vec{k}}_{\epsilon>0 \text{ Continuum states}}$$

$H\Psi(\vec{R},\vec{r}) = E\Psi(\vec{R},\vec{r})$ replaced by continuously infinite system of equations

## Discretized Continuum




By a Discretized-Continuum :

$$\Phi_{\vec{k}}(\vec{r}) \longrightarrow \Phi_{ilm}(\vec{r}) = r^{-1}u_{il}(r)Y_l^m(\Omega_r)$$

from $H_0\Phi_{ilm}(\vec{r}) = \epsilon_{il}\Phi_{ilm}(\vec{r})$

Reminder : projectile

$$H_0\Phi(\vec{r}) = \epsilon\Phi(\vec{r})$$

$$H_0 = T_r + V_{cf}(r)$$

$l \equiv$ projectile's orbital momentum

# 2. CDCC method [3/4]

Finite expansion (3-body) :

$$\Psi^{J}_{\mathrm{CDCC}}(\vec{R},\vec{r}) \approx \sum_i \Phi_{ilm}(\vec{r})\chi_i(\vec{R})$$

$$= (rR)^{-1} \sum_i^N \sum_{lL} Y^{JM}_{lL}(\Omega_r,\Omega_R) u_{il}(r) \chi^J_{ilL}(R)$$

To be determined

Angular function

2-body

$H\Psi = E\Psi$ replaced by Discrete finite system of equations

$$\left( -\frac{d^2}{dR^2} + \frac{L(L+1)}{R^2} + V^J_{ilL,ilL}(R) - \epsilon_i - E \right) \chi^J_{ilL}(R) + \sum_{i'l'L' \neq ilL} V_{ilL,i'l'L'}(R)\chi^J_{i'l'L'}(R) = 0$$

# 2. CDCC method [4/4]

The potential matrix elements

$$V^{J}_{ilL,i'l'L'}(R) = \left\langle Y^{JM}_{lL} r^{-1} u_{il} | V_{cT} + V_{fT} | Y^{JM}_{l'L'} r^{-1} u_{i'l'} \right\rangle$$

Numerical integration : $r$

Analytical integration : $(\Omega_r, \Omega_R)$

But

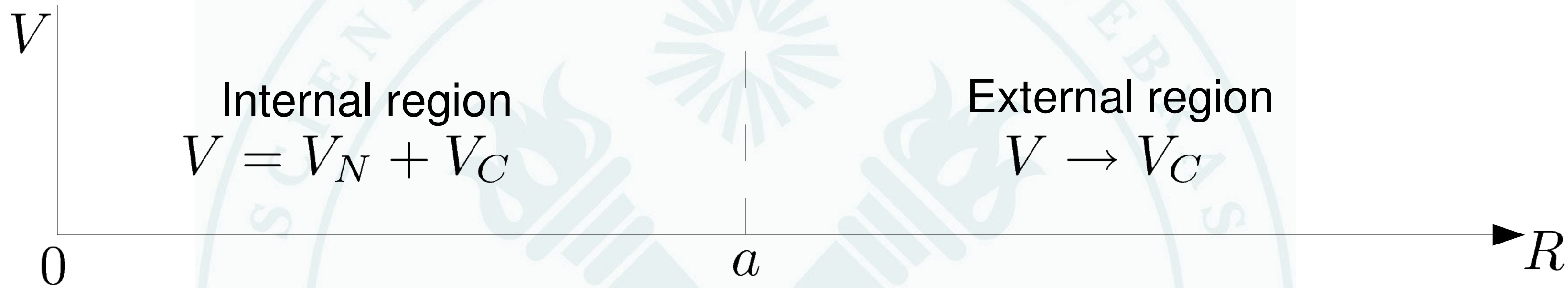
I. system of coupled equations (size up to 200)
II. numerical stability
III. processing time

**New tools to solve the system and to find** $\chi^{J}_{ilL}(R) \equiv \chi^{J}_{\gamma}(R)$

R-Matrix method and Lagrange mesh

# 3. The R-Matrix method

**Principle : space divided into two regions**




Internal : $r \leq a$

Nuclear + Coulomb interactions

$$\chi_{\gamma,\mathrm{int}} = \sum_{i=1}^{N} c_i \varphi_i(r)$$

$\varphi_i(r) \equiv N$ basis functions

External : $r > a$

Coulomb interaction only

$$\chi_{\gamma,\mathrm{ext}} \propto I_{\gamma'}(kr)\delta_{\gamma',\gamma} - U_{\gamma',\gamma} O_\gamma(kr)$$

Asymptotic behavior

Matching $\chi_{\gamma,\mathrm{int}}$ and $\chi_{\gamma,\mathrm{ext}}$ at $r = a$ provides
- Collision matrix $U_{\gamma,\gamma'}$
- Coefficients $c_i$

And the collision matrix $U_{\gamma,\gamma'}$ provide the cross sections

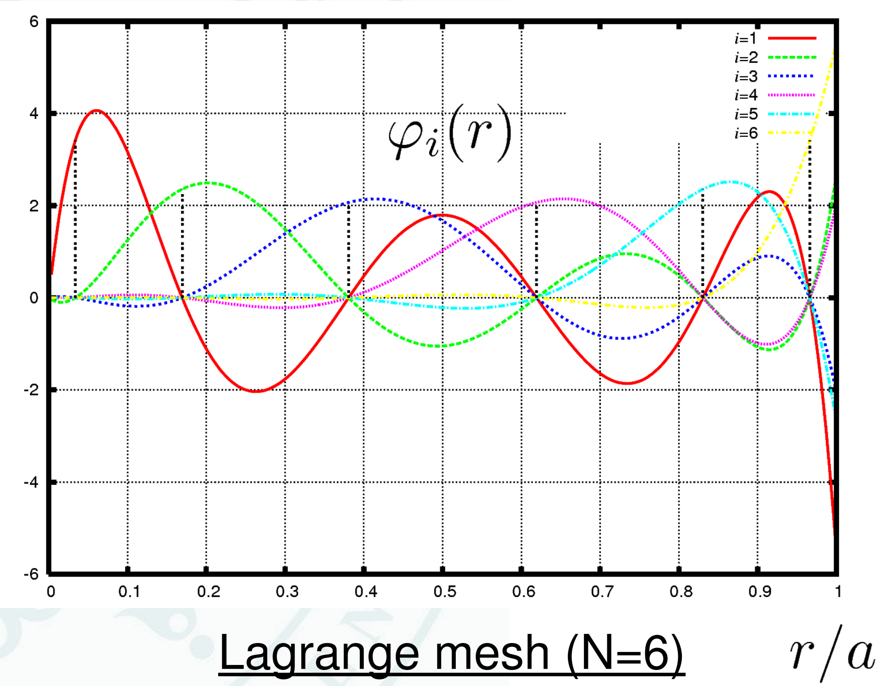
(results independent of $a$)

# 4. Lagrange mesh

To expand the radial wave function $\chi_{\gamma,\text{int}}(r)$

$$\chi_{\gamma,\text{int}}(r) = \sum_{i=1}^{N} c_i \varphi_i(r)$$

Use a Lagrange basis $\varphi_i(r)\ (\forall i = 1, \ldots, N)$ on the interval $[0, a]$




Lagrange mesh (N=6)

Properties :

$$\varphi_i(x_j) \propto \delta_{ij}$$

$$\int_0^a \varphi_i(r) V(r) \varphi_j(r) dr \approx V(x_i)\delta_{ij}$$

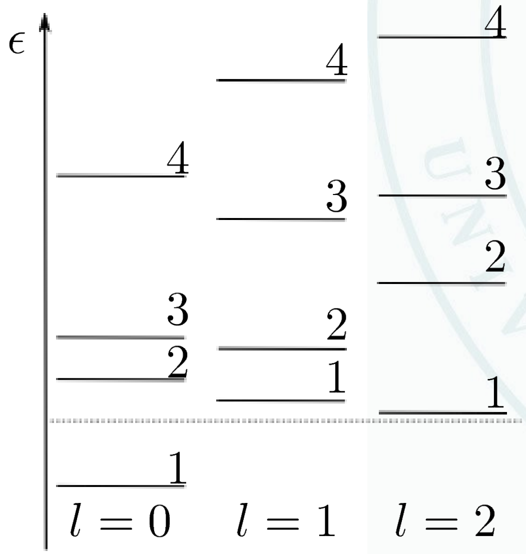
Advantages :

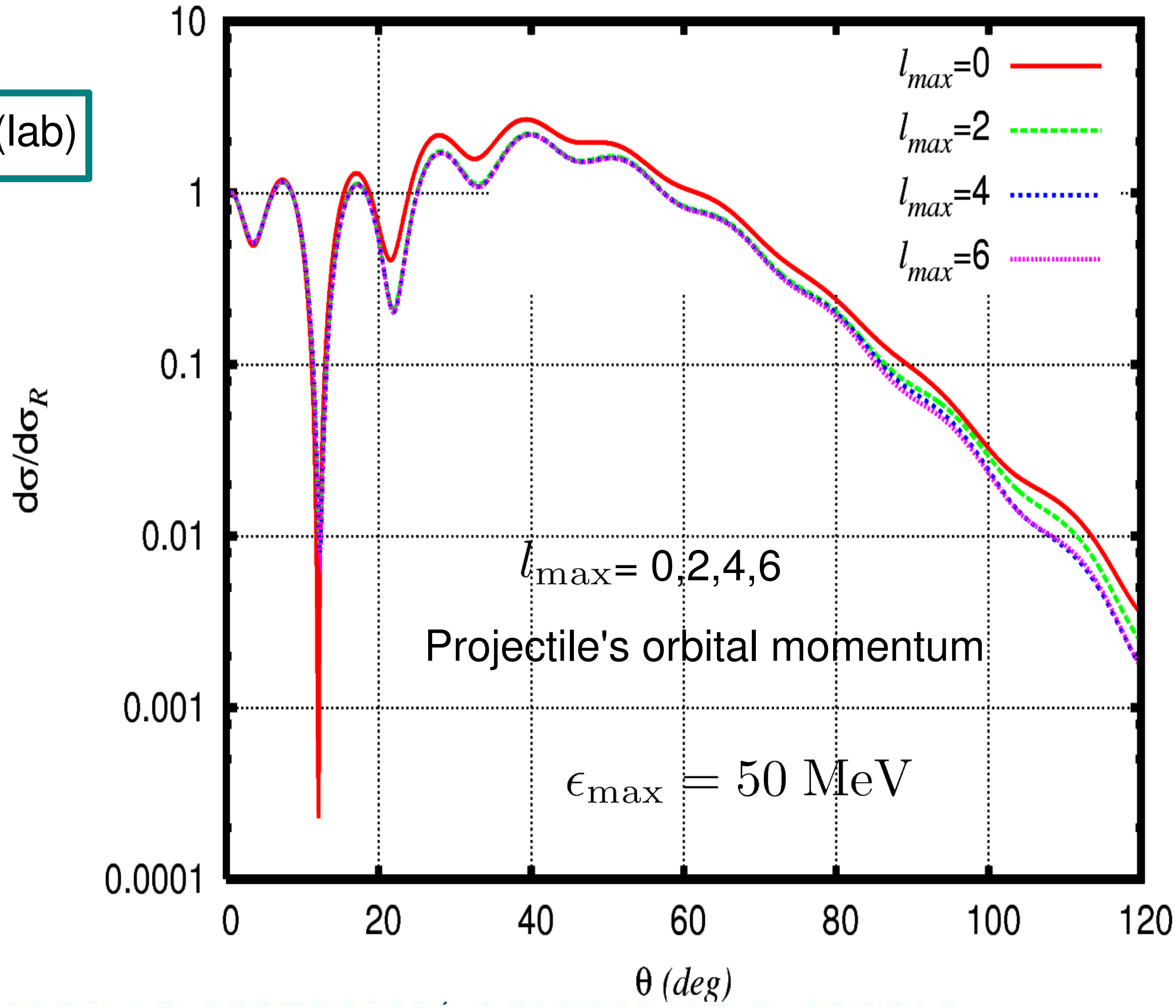
I. Potential needed at mesh points only $V(x_i) \quad \forall x_i \in [0, a]$
II. Matrix elements easy to compute
III. Relatively small $N\ (\sim 20 - 30)$
IV. Used in many problems (nuclear and atomic physics)

# 5. Numerical convergence [1/5]

d+$^{58}$Ni at $E_d$ = 80 MeV (lab)

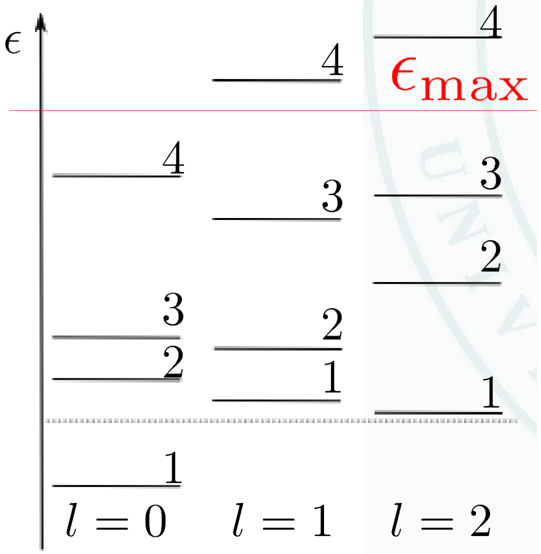
Discretized deuteron spectrum

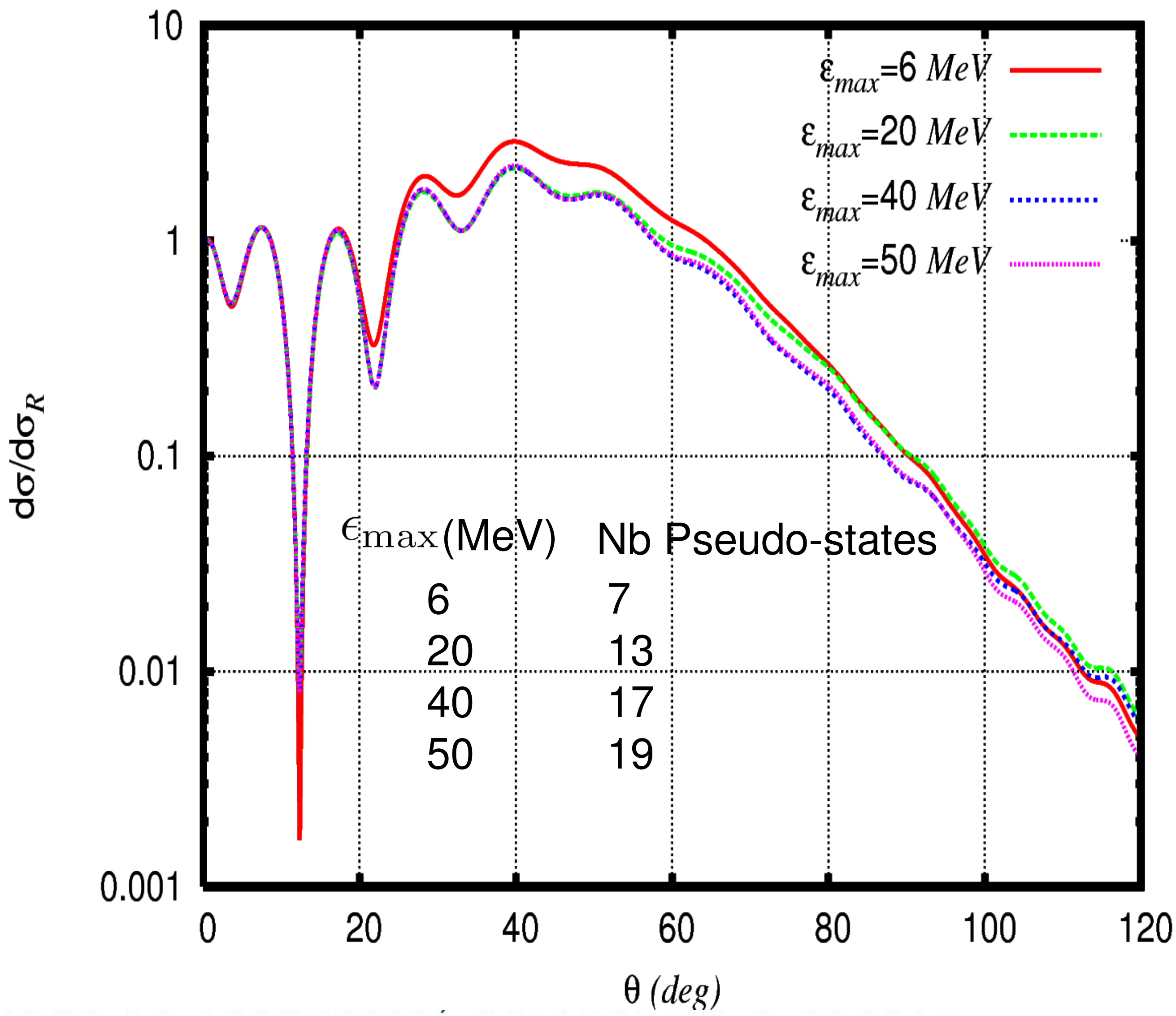


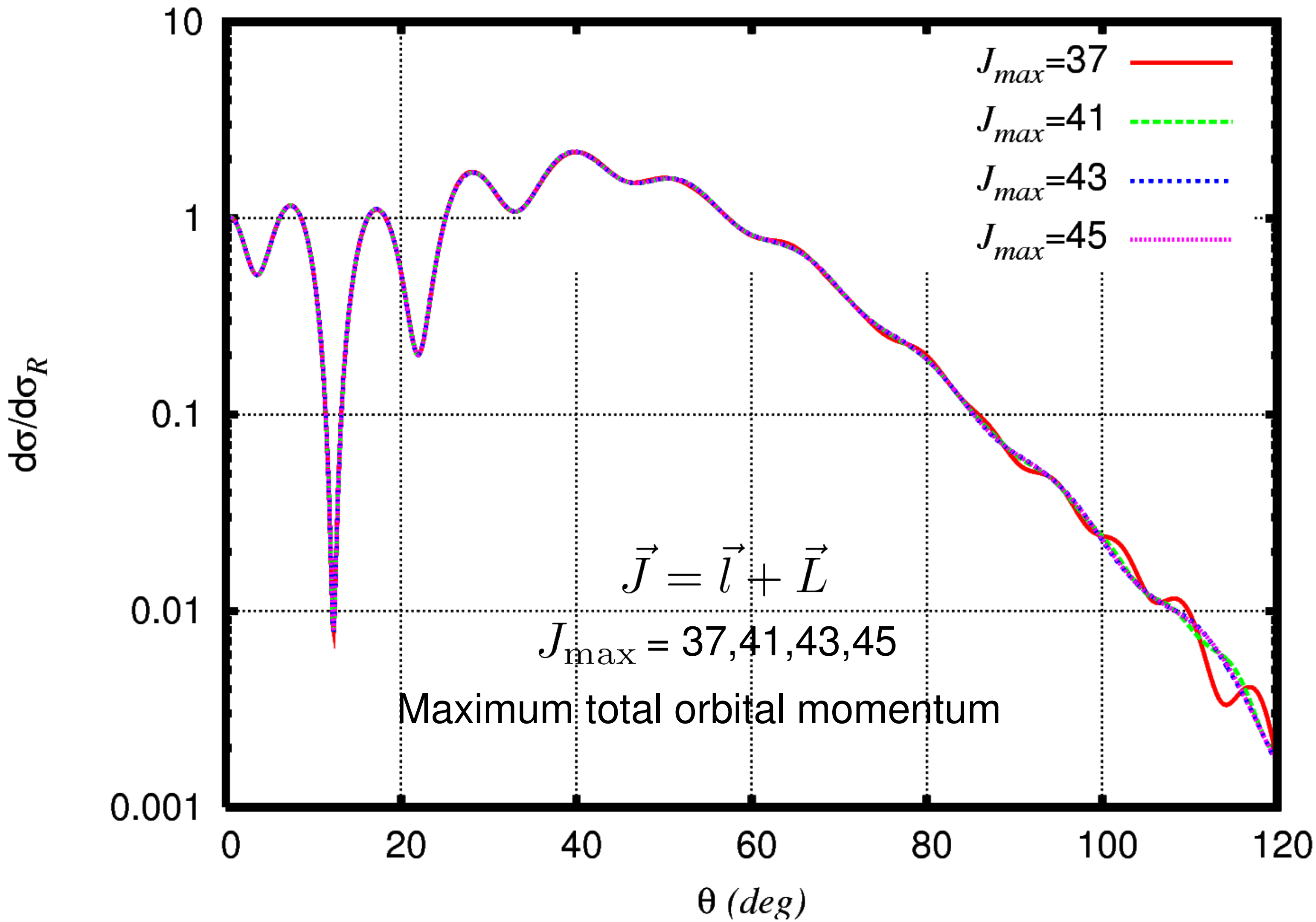
# 5. Numerical convergence [2/5]

Maximum energy
of Pseudo-states

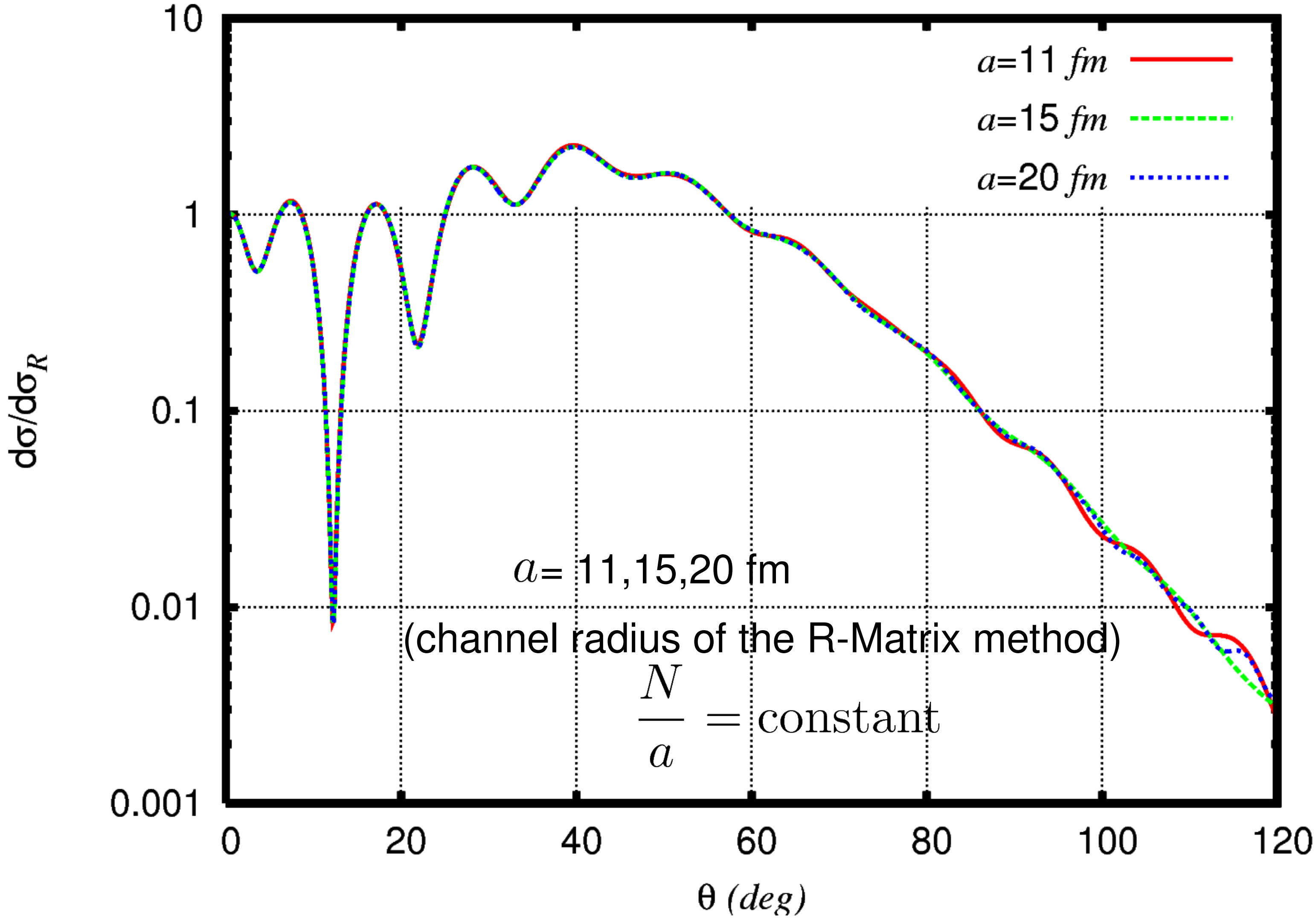


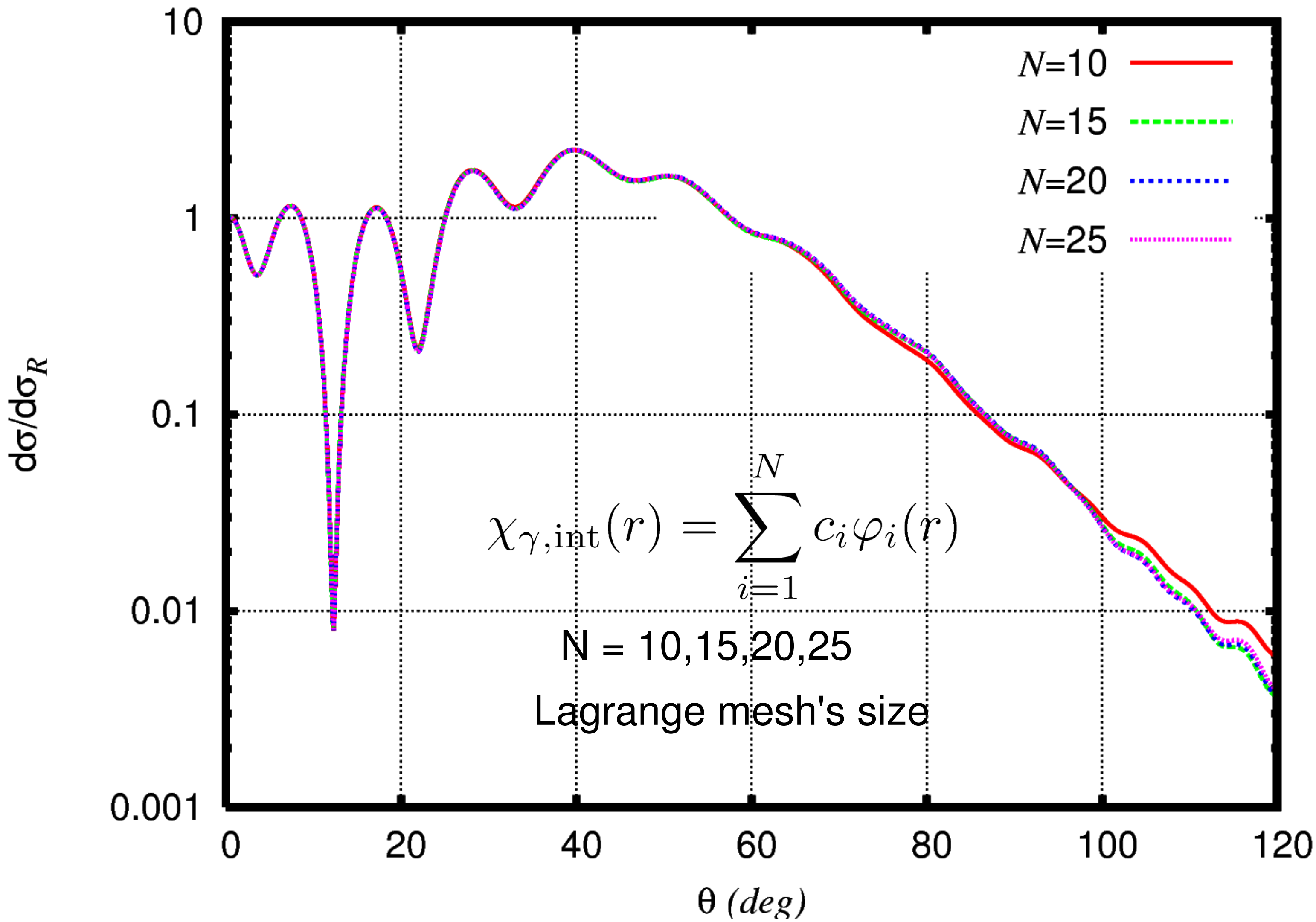
# 5. Numerical convergence [3/5]



# 5. Numerical convergence [4/5]



# 5. Numerical convergence [5/5]



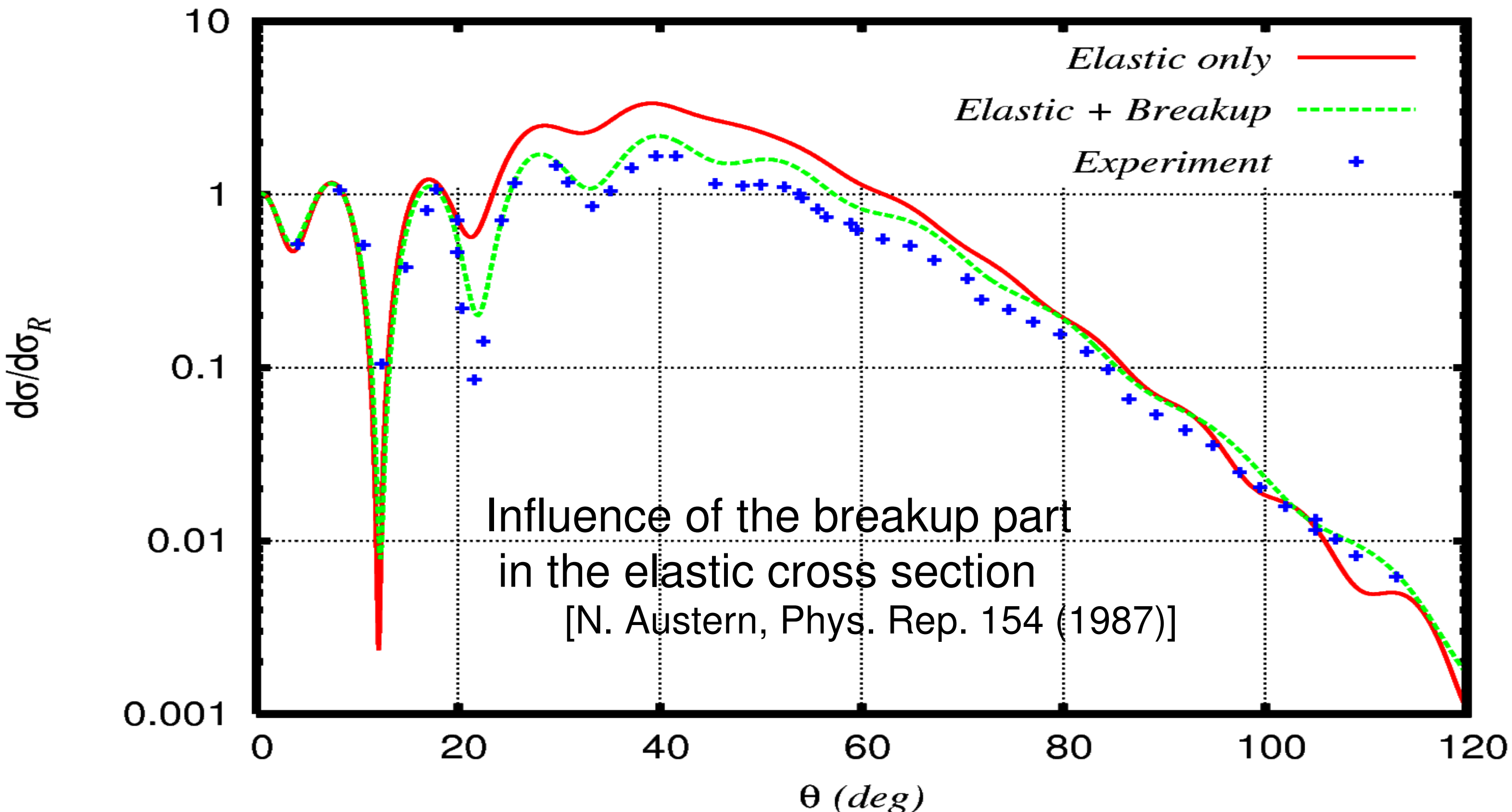
# 6. Comparison with experiment

[E. J. Stephenson, Phys. Rev. C 28 (1983)]

d+$^{58}$Ni at $E_d$ = 80 MeV (laboratory) Already done but with other methods

$V_{cf}$ = Minnesota potential [D. R. Thompson, Nucl. Phys A 286 (1977) ]

$V_{cT} + V_{fT}$ = from [F. D. Becchetti, Phys. Rev. 182 (1969)]



# 7. Conclusions

## Present

- Shorter computer times
- Influence of the breakup part in the elastic cross section

## Future

- Next steps
  - Take into account spin of the projectile
  - Discretization by another method called “bins”
- Breakup cross section
- Application with other nuclei : $^{7}Li$, $^{17}F$, $^{17}O$, $^{8}B$
- Future works : Extension to 3-body projectile (ex : $^{6}He$, $^{11}Li$)

Thank you for your attention !

# 9. Supplement

**Influence of the breakup part in the elastic cross section**

[N. Austern, Phys. Rep. 154 (1987)]

Global wave function

$$\Psi = \underbrace{\Psi_{\mathrm{d+Ni}}}_{\text{Elastic (+inelastic)}} + \underbrace{\Psi_{\mathrm{n+p+Ni}}}_{\text{Breakup}}$$

**Dependence on J of elastic cross section**

$$\frac{d\sigma}{d\Omega} = |f_{\mathrm{Coul}}(\Omega) + f_{\mathrm{add}}(\Omega)|^2$$

with $f_{\mathrm{add}}(\Omega) = \frac{1}{2ik} \sum_{J=0}^{\infty} (2J+1) \exp\left(2i\sigma_J\right)(U^J_{\gamma,\gamma'} - 1) P_J(\cos(\theta))$

where $\sigma_J - \sigma_0 = \sum_{n=1}^{J} \mathrm{atan}(\frac{\eta}{n})$